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Predictions of angle dependent tortuosity and elasticity effects on sound propagation in cancellous bone

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The anisotropic pore structure and elasticity of cancellous bone cause wave speeds and attenuation in cancellous bone to vary with angle. Previously published predictions of the variation in wave speed with angle are reviewed. Predictions that allow tortuosity to be angle dependent but assume isotropic elasticity compare well with available data on wave speeds at large angles but less well for small angles near the normal to the trabeculae. Claims for predictions that only include angle-dependence in elasticity are found to be misleading. Audio-frequency data obtained at audio-frequencies in air-filled bone replicas are used to derive an empirical expression for the angle-and porosity-dependence of tortuosity. Predictions that allow for either angle dependent tortuosity or angle dependent elasticity or both are compared with existing data for all angles and porosities. © *2009 Acoustical Society of America.* [DOI: 10.1121/1.3242358]

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I. INTRODUCTION

Clinical detection of osteoporosis involves measurement of broadband ultrasonic transmission at peripheral sites containing cancellous bone, which has a highly porous anisotropic cellular network structure filled with fatty bone marrow and including calcified plate-like elements known as trabeculae. The inclinations of the trabeculae vary with the site in the body, possibly as a consequence of mechanical requirements, for example, being somewhat random in the femoral head but more aligned in the calcaneous. Although typical clinical measurements are made normal to the trabeculae, the anisotropic structure of trabecular bone causes wave properties to vary with direction (Hosakawa and Otani, 1998; Hughes *et al.*, 1999; Lee *et al.*, 2007). Some success in modeling sound transmission in cancellous bone has been achieved by means of various forms of Biot theory (Biot 1956a, 1956b) which predicts two types of compressional wave (known as "fast" and "slow") and a shear wave. A basic premise of Biot theory is that the incident sound wavelengths are significantly larger than typical microstructural dimensions. Since the initial application of Biot theory to sound propagation in bone (McKelvie and Palmer, 1991), there has been considerable debate concerning the validity of this application. According to Williams (1992), the pore sizes in cancellous bone vary between 0.5 and 1 mm: a similar range of pore diameters is quoted in Hughes *et al.*, 2003. Also according to Williams (1992), the wavelength of the fast wave in water-saturated cancellous bone at 0.5 MHz is stated to lie between 5 and 7 mm for porosities between 0.1 and 0.4. This corresponds to fast wave speeds of between 2500 and 3500 m/s. In the frequency range from 1 kHz and 1 MHz, Hughes *et al.* (2003) predicted fast wave speeds of between 3700 and 5000 m/s for both water-filled and marrow-filled bones. The higher wave speeds will correspond to wavelengths on the order of 10 mm. In a similar frequency range, Hughes *et al.* (2003) predicted slow wave speeds of approximately 1500 m/s corresponding to wavelengths of between 1.5 m at 1 kHz and 1.5 mm at 1 MHz. Consequently, except at frequencies greater than 1 MHz, the predicted wavelengths in cancellous bone are an order of magnitude greater than the pore size and Biot theory should be applicable. At frequencies higher than 1 MHz, the slow wave should be subjected to a significant degree of scattering and, thereby, there should be higher transmission loss than predicted by Biot theory. However, even if Biot theory underestimates the attenuation of the frequency components of

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a slow wave pulse above 1 MHz, the influence on predicted waveforms will be small since the bone will act as a low pass filter and the lower frequency content will be more important.

Using isotropic Biot–Allard theory (Allard 1993), Fellah *et al.* (2004) find that tortuosity, defined as the ratio of the average length of the flow path through a porous medium sample to the thickness of the sample, plays an important role in propagation through cancellous bone since it affects the inertial coupling between fluid and solid. The theory employed by Fellah *et al.* (2004) introduces a viscous characteristic length Λ, due originally to Johnson *et al.* (1987), instead of the pore shape parameter originally used by Biot (1956a, 1956b) and, subsequently, by Hughes *et al.* (2007) and Lee *et al.* (2007). The viscous characteristic length depends on the narrowest pore sections where the effects of viscous drag are greatest. Fellah *et al.* (2004) predict that the viscous characteristic length may also have an important influence on wave transmission through bone but less than that of tortuosity.

To model the effects of the anisotropy of cancellous bone, Hughes *et al.* (2007) developed a stratified-Biot (SB) theory. They assumed an idealized microstructure of periodic parallel plates representing the trabeculae. The direction perpendicular to the plate axes, i.e., the dominant structural orientation, was taken to correspond to the zero value for the incidence angle. The resulting theory while giving reasonable agreement with data for large angles ($>30°$) from the normal to the predominant trabeculae direction is found to over-predict the fast wave speed at low angles ($<30°$) and to underestimate the slow wave speed at all angles. Hughes *et al.* (2007) also considered the influence of anisotropic Young's modulus. However, their development results only in a slight improvement in predictions compared with SB theory.

Lee *et al.* (2007) modeled the influence of angle-dependency in the elastic properties on sound propagation in cancellous bone. They considered two formulations of Biot theory and claimed that both give good agreement with data for the variation in fast wave speed with angle and porosity. However, agreement with comparable data for slow wave speeds was less good. Neither of the approaches used by Lee *et al.* (2007) includes an angle dependent tortuosity. Specifically, their tortuosity includes porosity-dependence but exclude angle-dependence, i.e., Lee *et al.* (2007) introduced anisotropy entirely through the elastic properties and ignore the effects of anisotropy in the pore structure. As shown in Fig. 5 of Hughes *et al.* (2007), an angle dependent tortuosity alone can explain some of the variation in fast wave speed with porosity and angle that has been observed. Moreover, unfortunately, in their paper Lee *et al.* (2007) compared predictions for the porosity of 0.65 with data for a porosity of 0.77.

Here, the heuristic form of angle dependent elasticity suggested by Lee *et al.* (2007) is combined in Biot–Allard theory with a heuristic angle and porosity dependent tortuosity function based on data obtained at audio-frequencies with air-filled (human) bone replicas by Attenborough *et al.* (2005). The replicas were 13 times real scale. However, the incident pulses were centered on 1 kHz, so the long wavelength condition for application of Biot theory is easily satisfied. The assumed form of angle-dependence is consistent with the observation that the fast wave speed increases with angle from the normal to the trabeculae. Predictions are explored that (a) only allow for angle dependent tortuosity, (b) only allow for angle dependent elasticity, and (c) allow for both. The predicted angle dependent phase velocities of fast and slow waves are compared with data for bovine bone (Hughes *et al.*, 1999).

II. THEORY

The Biot–Allard model for waves in fluid-saturated poro-elastic media (Allard, 1993) allows for thermal exchange and viscous drag between pore-fluid and the solid framework by introducing two characteristics lengths: the viscous (Λ) and thermal (Λ') characteristic lengths related to pore form factors c and c' by the following relationships:

$$\Lambda = \frac{1}{c}\left(\frac{8\alpha_\infty \eta}{\phi\sigma}\right)^{1/2}, \quad \Lambda' = \frac{1}{c'}\left(\frac{8\alpha_\infty \eta}{\phi\sigma}\right)^{1/2}, \tag{1}$$

where ϕ is porosity, and σ is the flow resistivity (which is equal to η, the dynamic viscosity coefficient, divided by permeability).

Thermal exchange effects between solid and fluid are included through a frequency-dependent bulk modulus of the fluid. This is calculated using (Allard, 1993)

$$K_f(\omega) = \frac{\gamma K_f}{\gamma - (\gamma-1)\left[1 + \dfrac{8\eta}{j\Lambda'^2 B^2 \omega\rho_0}\left(1 + j\rho_0 \dfrac{\omega B^2 \Lambda'^2}{16\eta}\right)^{1/2}\right]^{-1}}, \tag{2}$$

where γ is the fluid specific heat ratio, B^2 is the Prandtl number, and K_f is the isothermal bulk modulus of the fluid. Thermal effects, while fairly important in air-filled porous materials, are expected to be of minor importance in marrow-filled bone. As yet, values for the characteristic lengths in bone have not been evaluated directly. However, Sebaa *et al.* (2006) found that values of Λ between 8 and 10.5 μm are consistent with data. For certain idealized pore structures, it is known that $c' \sim c/2$ (Allard, 1993).

The dependence of tortuosity on angle and porosity assumed by Hughes *et al.* (2007) is given by

$$\alpha_\infty(\theta) = 1 + \left[\frac{(1-\phi)\rho_s}{\langle\rho\rangle}\right]\cot^2 \theta, \tag{3}$$

where $\langle\rho\rangle = \phi\rho_f + (1-\phi)\rho_s$, ρ_f and ρ_s being the mass densities of the fluid and solid, respectively, and ϕ is the porosity. This idealized angle-dependence implies infinite tortuosity for $\theta = 0°$ when sound travels normal to the parallel plates in the assumed parallel plate microstructure and a value depending on the relative densities of solid and fluid for $0° < \theta < 90°$. The tortuosity defined by Eq. (3) would be unity for propagation parallel to the plates if the plates are rigid; i.e., Equation (3) has an angle-dependence similar to that of the tortuosity in an idealized microstructure of parallel cylindrical pores in a rigid frame. In such a medium, the tortuosity

TABLE I. Properties and r [Eq. (4)] values for bone replicas (Attenborough *et al.*, 2005).

Replica type	Porosity	r
Iliac crest	0.8386	0.888
Femoral head	0.7426	0.591
Lumbar spine (LS2)	0.9173	0.521
Calcaneus	0.8822	0.816
Lumbar spine (LS4)	0.9121	0.259

would be given by $1/\sin^2(\theta)=\mathrm{cosec}^2(\theta)=1+\cot^2(\theta)$ where $\theta=0°$ is normal to the pore direction.

Cancellous bone microstructure departs significantly from either parallel plate or parallel pore idealizations. There is no evidence of values of tortuosity higher than 2.64 in the bone (see Table I in Hughes *et al.*, 2007). So the function given by Eq. (3) is least likely to be reliable for low angles, precisely where Hughes *et al.* (2007) found the biggest discrepancies between SB theory and data. According to the geometrical interpretation of tortuosity, it is determined entirely by the pore structure, is independent of the saturating fluid, and is independent of scaling. Consequently, extreme values of the angle-dependence of tortuosity may be derived empirically by referring to the average measured tortuosity values deduced from audio-frequency measurements on five air-filled stereo-lithographical cancellous (human) bone replicas at 13 times actual scale (Attenborough *et al.*, 2005). These data show that cancellous bone microstructure has orthotropic anisotropy. It is assumed that the dependence of tortuosity on porosity is given by (Berryman, 1980):

$$\alpha_\infty = 1 - r\left(1 - \frac{1}{\phi}\right), \tag{4}$$

where r is a variable calculated from a microscopic model of a frame moving in a fluid. The values of r required for consistency with the values of tortuosity for $\theta=0°$ deduced from the acoustical measurements on air-filled replica bones of known porosity (Attenborough *et al.*, 2005) are listed in Table I.

A heuristic form for porosity and angle dependent tortuosity may be written as

$$\alpha_\infty = 1 - r\left(1 - \frac{1}{\phi}\right) + k\cos^2(\theta), \tag{5}$$

where r and k can be considered adjustable. The assumed angle-dependence function is chosen arbitrarily but is simple and consistent with the expected variation in fast wave speed with angle. It should be noted that, if tortuosity has angle-dependency, as in Eq. (5), then so do the characteristic lengths and form factors [through Eq. (1)]. A range of possible values of r and k have been found by comparing predictions of Eq. (5) for $\theta=0°$ and 90°, respectively, with values deduced from air-filled replica bones (Attenborough *et al.*, 2005) of known porosity. Values of r and k are found by solving the resulting simultaneous equations. The angle dependent function representing the extremes of tortuosity measured in the bone replicas is

TABLE II. Default input parameters of the anisotropic Biot–Allard model for cancellous bone.

Parameters	Value
Density of solid bone, ρ_s	1960 kg/m^3
Density of fluid, ρ_f	1000 kg/m^3
Young's modulus of bone, E_s	20 GPa
Bulk modulus of fluid, K_f	2.2 GPa
Poisson's ratio of solid, υ_s	0.32
Poisson's ratio of frame, υ_b	0.32
Porosity, ϕ	0.65
Power index, n	$1.23\sin^2(\theta)+2.35\cos^2(\theta)$
Viscosity of fluid, η	0.001 Pa s
Permeability, k_0	5×10^{-9} m^3
Frequency, f	1 MHz
Fluid specific heat ratio, γ	1.0107
Prandtl number, B^2	7
Form factor, c	1
Form factor, c'	$c/2$

$$\alpha_\infty = 1.025 + 0.864\cos^2(\theta). \tag{6}$$

Williams (1992) suggested that the dependences of skeletal frame moduli (Young's modulus E_b, bulk modulus K_b, and rigidity modulus μ_b) in terms of bone volume fraction $(1-\phi)$ and Young's modulus of the solid material of the frame (E_s) are given by

$$E_b = E_s(1-\phi)^n, \tag{7a}$$

$$K_b = E_b/(1-2\upsilon_b), \tag{7b}$$

$$\mu_b = E_b/(1+2\upsilon_b), \tag{7c}$$

where the exponent n varies from 1 to 3 according to Gibson (1985), depending on the angle (θ) with respect to the dominant structural orientation (of the trabeculae, for example) according to

$$n = n_1\sin^2(\theta) + n_2\cos^2(\theta). \tag{8}$$

Values of $n_1=1.23$ and $n_2=2.35$ are chosen by Lee *et al.* (2007) to be consistent with the work of Williams (1992). Default values of the parameters required by the anisotropic Biot–Allard theory are listed in Table II. As remarked earlier, neither of the theoretical approaches used by Lee *et al.* (2007) includes an angle dependent tortuosity. They used a porosity dependent but angle independent tortuosity in two different formulations of Biot theory.

III. COMPARISONS WITH DATA

Figure 1 compares predictions of anisotropic Biot–Allard theory based on Eqs. (5) and (8) with data obtained on bovine femur by Hughes *et al.* (1999). The predictions include angle dependent tortuosity by allowing θ to vary in Eq. (5) but assume isotropic elasticity by setting $\theta=90°$ in Eq. (8). These predictions are similar to those of the stratified-Biot model in Hughes *et al.*, 2007 (see their Fig. 5) but assume a less extreme variation of tortuosity with angle.

Figure 2 compares predictions that allow angle-dependency in both tortuosity and elasticity with the same

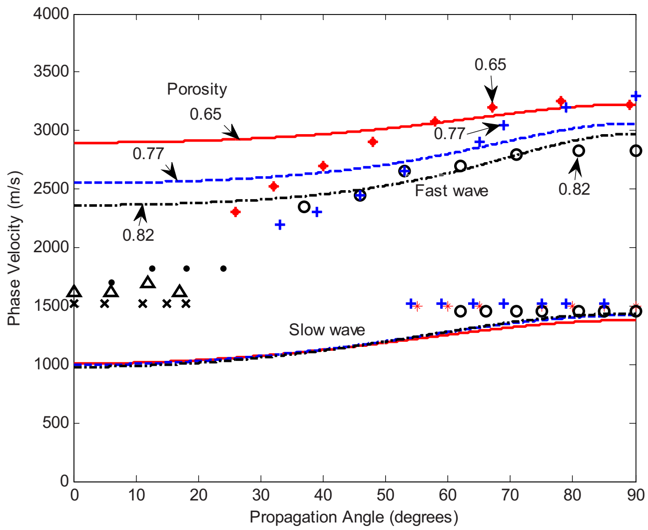

FIG. 1. (Color online) Hughes *et al.* (1999) data (symbols +, ○, and *) for three "parallel" samples on wave speeds as a function of angle (for porosities of 0.65, 0.77, and 0.82), and data (symbols ×, Δ, and ·) for three "perpendicular" samples compared with predictions (lines) assuming an angle and porosity dependent tortuosity function [Eq. (5) with r=0.259 and k=0.864] and isotropic elasticity [Eqs. (7) and (8) and Table II with n_1=1.23 and θ=90°].

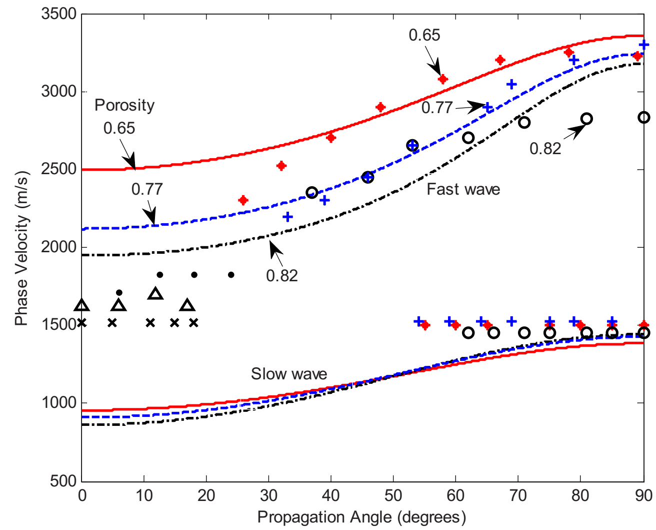

FIG. 3. (Color online) Data (symbols +, ○, and *; ×, Δ, and ·) corresponding to porosities of 0.65, 0.77, and 0.82, for wave speeds as a function of angle compared with predictions (lines) assuming an angle and porosity dependent tortuosity function [Eq. (5) with r=0.259 and k=0.864] and angle dependent elasticity [Eqs. (7) and (8) and Table II with n_1=1.15 and n_2=1.6].

data using Eqs. (5), (7), and (8) with r=0.047 and k=0.864. The resulting predictions are rather similar to those in Lee *et al.*, 2007 (see their Fig. 2). However, it should be noted that Lee *et al.* (2007) compared predictions for porosity of 0.65 with data for a porosity of 0.77. Although, as they asserted, the overall prediction of angle-dependence is improved through use of Eq. (8), it is at the cost of accuracy in the predicted porosity-dependence. In short, the predictions by Lee *et al.* (2007) of the influence of porosity on angle-dependence are not as good as they claim.

It should be noted that although the predictions of fast wave speeds in Fig. 2 are very similar to those in Fig. 2 of Lee *et al.* (2007), use of Eq. (5) rather than the fixed values

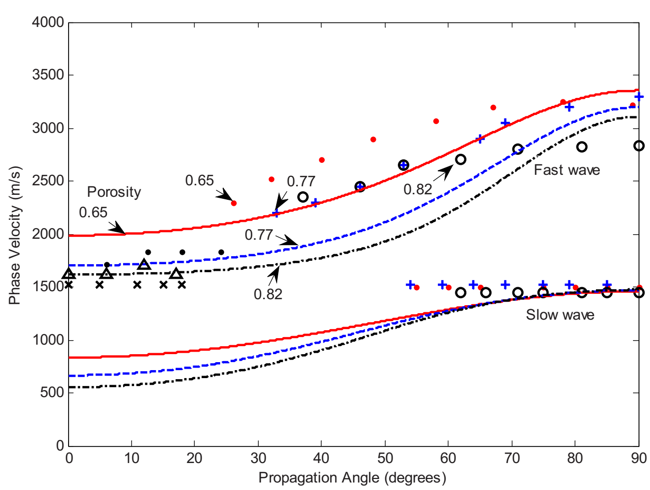

FIG. 2. (Color online) Data for three "parallel" (symbols +, ○, and *) samples and three "perpendicular" (symbols ×, Δ, and ·) samples (for porosities of 0.65, 0.77, and 0.82) on wave speeds as a function of angle compared with predictions (lines) assuming an angle and porosity dependent tortuosity function [Eq. (5) with r=0.047 and k=0.864] and angle dependent elasticity [Eqs. (7) and (8) and Table II].

of tortuosity used by Lee *et al.* (2007) means that the slow wave predictions at large angles are slightly improved compared with those in Lee *et al.*, 2007.

To obtain improved agreement between predicted and measured fast wave speeds over all angles when including both angle dependent tortuosity and elasticity in the predictions, the dependence on angle in Eq. (8) must be reduced. This means that the coefficient values n_1 and n_2 in Eq. (8) should be reduced. An example result, which confirms that, thereby, an improved prediction of porosity and angle-dependence can be obtained, is shown in Fig. 3. The values of the coefficients n_1 and n_2 have an important effect on the phase velocities of fast and slow waves, especially at low angles. Reducing the values of n_1 and n_2 increases the predicted phase speed of the fast wave particularly at low angles.

Lee *et al.* (2007) also compared predictions and data for wave speeds at 1 MHZ in directions perpendicular to and parallel with the dominant structural orientation. The corresponding predictions from Eqs. (5), (7), and (8) are shown in Fig. 4.

The value of r is predicted to have important influence on the fast wave speed variation with porosity perpendicular to the dominant structural orientation and on the slow wave speed variation with porosity parallel to the dominant structural orientation. Although not shown here, the value of n_1 is predicted to have an important influence on the fast wave speed variation with porosity, parallel to the dominant structural orientation. Other calculations suggest that an angle dependent viscous characteristic length has potentially important effects on the variation in slow wave speed with porosity for measurements close to the dominant structural direction.

IV. CONCLUSION

Previous work on the influence of anisotropic pore structure and elasticity in cancellous bone has been extended by

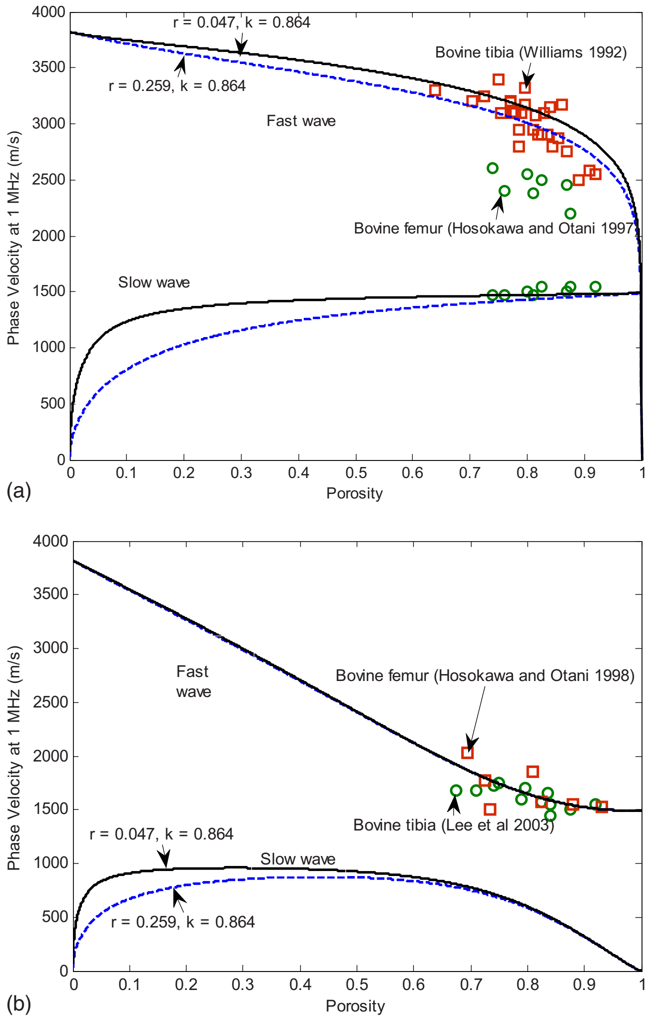

FIG. 4. (Color online) Predictions (lines) and data (symbols) for porosity-dependence of wave speeds (a) for propagation perpendicular to the dominant structural orientation direction assuming a porosity dependent tortuosity function [Eq. (5) with $\theta=90°$, values of r and k as labeled] and anisotropic elasticity given by Eqs. (7) and (8) and Table II with $\theta=90°$; (b) for propagation parallel to the dominant structural orientation assuming a porosity dependent tortuosity function [Eq. (5) with $\theta=0°$, values of r and k as labeled] and elasticity given by Eqs. (6) and (7) with $\theta=0°$, and parameter values in Table II.

developing an anisotropic Biot–Allard model allowing for angle dependent tortuosity and elasticity. The extreme angle-dependence of tortuosity corresponding to the parallel plate microstructure used by Hughes *et al.* (2007) has been replaced by angle dependent tortuosity values based on data for slow wave transmission through air-filled bone replicas. It has been shown that the good agreement claimed by Lee *et al.* (2007) using only angle dependent elasticity is misleading and that more complete predictions allowing for angle-dependency in both tortuosity and elasticity have greater validity. Although agreement with data even after adjustment of the parameter values for angle dependent elasticity used by Lee *et al.* (2007) is not particularly good, the anisotropic Biot–Allard model will be useful to give further insight into the factors that have the most important influence on the angle-dependency of wave speeds and attenuation in cancellous bone.

ACKNOWLEDGMENT

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